Review For Exam 3

The directions for the exam are as follows:

"WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!"

1. The exam consists of 10 core problems and 2 extra-credit problems. If you wish, you can do all the 12 problems, but your score will only add up to 100 points. Partial credit will be given.

2. You are allowed to use a scientific calculator. Don't forget to bring it

3. When you are studying for this exam, be sure to work through sections that you know least of all first.

4. Odd exercises have solutions at the back of your textbook.

Warning! Be sure to work on ALL exercises below.

Advanced Convergence/ Divergence Practice Problems

- The problems below are designed to help you become more comfortable with the various convergence tests and at the same time review previously learned material. They are not essential if you simply want to do well on my test. It is recommended that you do all the regular problems on this review section before attempting the problems that are listed here.

(a) Find the exact sum of the series $\sum_{n=1}^{\infty} \frac{2^n}{b_n}$ where $b_n = \int_0^{\infty} y^n e^{-y}\, dy$.

(b) For what values $\alpha \in \mathbb{R}$, if any, does the series $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\alpha}}$ converge?

(c) Find the exact sum of $\sum_{n=0}^{\infty} r^n$ where $r = \dfrac{1}{\int_0^{\pi/3} \frac{\tan^3 x}{\cos^3 x}\, dx}$.

(d) Determine if $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\ln n}}$ converges.

(e) Determine if $\sum_{n=1}^{\infty}\left(\sqrt[n]{2}-1\right)$ converges.

(f) Determine if $\sum_{n=1}^{\infty}\left(\sqrt[n]{2}-1\right)^3$ converges.

(g) Determine if $\sum_{n=2}^{\infty}\frac{1}{n+n\cos^2 n}$ converges.

(h) Determine if $\sum_{n=1}^{\infty} Sin\left(\frac{1}{n}\right)$ converges.

(i) Find the exact value of $\prod_{n=1}^{\infty}\frac{\tan^{-1}(n+1)}{\tan^{-1}(n)}$.

(j) Find the limit of the sequence $a_n=\int_0^9 (x-1)^{-\frac{1}{n}}dx$ or prove that the limit does not exist.

(k) Find the limit of the sequence $a_n=\int_0^{\pi/2} e^{-nx}\cos x dx$ or prove that the limit does not exist.

(l) Find the limit $\lim_{n\to\infty}\sqrt[n]{n!}$

Section 8.1

- Be able to compute limits of elementary sequences (**P. 440 Exercises 5-33 [odd]**)
- Determine which sequences are bounded and monotonically increasing/ decreasing (**P. 440 Exercises 37-40**)

Section 8.2

- Determine the convergent series and find their sum (**P. 449 Exercises 7-27 [odd]**)
- Express the number as a ratio of integers (**P. 449 Exercises 29-34**)
- **Possible Extra-Credit:** (**P. 450-451 Exercises 45-60**)

Section 8.3

- Be able to use p-series, comparison, or limit comparison tests to determine whether given series is convergent (**P. 459 Exercises 7-31 [odd]**)
- **Possible Extra-Credit:** (**P. 460 Exercises 45-48**)

Section 8.4

- Be able to apply alternating series test, ratio, and root tests, as well as absolute convergence to determine convergent series. (**P. 469 Exercises 19-39 [odd]**)

Section 8.5

- Know how to find the radius and interval of convergence of a given power series (**P. 475 Exercises 3-21 [odd]**)
- **Possible Extra-Credit:** (**P. 475 Exercises 23-24**)
- **Possible Extra-Credit:** Let $p = \prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)$. For which values of x does $\sum_{k=1}^{\infty}\frac{x^k}{k^p}$ converge?
- Suppose that the series $\sum_{n=0}^{\infty} b_n x^n$ converges for $|x| < 2$ by the root test.

  What can you say about the radius of convergence of $\sum_{n=0}^{\infty}(b_n)^2 x^n$?

Section 8.6

- Find the power series representation (**P. 480, Exercises 3-9, 15-19 [odd]**).
- Evaluate the indefinite integral as a power series. (**P. 752, Exercises 25-28**).
- **Possible Extra-Credit:** Prove or disprove. If *f* has a power series representation $\sum_{n=0}^{\infty} a_n x^n$ in the interval (-r, r), then *f* has derivatives of all orders in the interval (-r, r).

Section 8.7

- Be able to compute Taylor series for the following functions:

  (a) $f(x) = e^x$

  (b) $f(x) = \cos x$

  (c) $f(x) = \sin x$

  (d) $f(x) = \ln x$

  (e) $f(x) = tan^{-1}x$

- Be able to use your knowledge of basic Taylor series to quickly compute power series of similar functions.

(a) $f(x) = xe^{x}$

(b) $f(x) = \sin(6x^2)$

(c) $f(x) = 3x tan^{-1}(x^3)$

- Find the Taylor Series. (**P. 493, Exercises 11-17 [odd]**).
- Evaluate the indefinite integral as an infinite series (**P. 494, Exercises 43-45 [odd]**)
- Be able to use infinite series to solve limits (**P. 494, Exercises 51-53**)
- Find the sum of the series (**P. 494, Exercises 59-64**).
- **Possible Extra-Credit:** Prove or disprove. If *f* has a power representation $\sum_{n=0}^{\infty} a_n x^n$ in the interval (-r, r), then *f* has a Maclaurin series representation in (-r, r) with $\frac{f^{(n)}(0)}{n!} = a_n$.
- **Possible Extra-Credit:** Let $f(x) = x\tan^{-1}(x)$. Calculate the 100th derivative of *f* at x = 0. That is, find $f^{(100)}(0)$.
- **Possible Extra-Credit:** If *f* is a function, for which the power series $\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$ can be computed in some interval (-r, r), does it follow that $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$ for all x in some subinterval of (-r, r)? [Hint: Try $f(x) = \begin{cases} e^{-x^{-2}} & x \neq 0 \\ 0 & x = 0 \end{cases}$. Compute the Maclaurin series of *f*. Does *f* equal to its Maclaurin series in some interval (-r, r)?]
- **Possible Extra-Credit:** Let *a* > 0. Use power series to find a function *f* satisfying *f ''(x) = -af(x)* where *f (0) = 0* and *f ' (0) =* $\sqrt{a}$.
- **Possible Extra-Credit:** Use the identity $e^{i\theta} = \cos\theta + i\sin\theta$ to find the identities for $\cos 3\theta$ and $\sin 3\theta$ in terms of $\cos\theta$ and $\sin\theta$.
- **Possible Extra-Credit:** Suppose that *f* (*x*) is continuous, but nowhere differentiable on $(-\infty, \ \infty)$ (there are such functions. Trust me!). Then, by the fundamental theorem of calculus, $F(x) = \int_0^x f(t)dt$ is differentiable with derivative *f* (*x*). Does *F* have a power series expansion about *x* = 0? How about some other point?